Significant Figures and Uncertainty in Measurements

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When performing measurements and calculations how many digits do we extend to in our final answers? This can be explained by using significant figures to express the uncertainty in measurements. Significant figures is a tool that helps determine the number of digits used in a measurement. When performing measurements and calculations, significant figures express the uncertainty in a measurement and determine how many digits are used in the final answer.

The general rules for determining significant figures are as follows:

Any nonzero digit	123, 1.23, .123 all have 3 significant figures.
A zero that is between two nonzero digits	1023, 1.023, .1023 all have 4 significant figures.
Zeros that occur AFTER the decimal place are significant	123.0 has 4 significant figures. 123.00 has 5 significant figures.
All zeros to the left of the first nonzero digit are NOT significant	.00123 has 3 significant figures.
Zeros that occur without a decimal are NOT significant	1230 has 3 significant figures

Significant Figures in Multiplication

When multiplying and dividing numbers, the number of significant figures in your answer will be equal to the number with the *least* significant figures. Let's divide 123 by 3.14159 which equals 39.152149071. In this example, 123 has 3 significant figures and 3.14159 has 6. Therefore the answer can only have 3 significant figures.

$$\frac{123}{3.14159}=39.1(3\ \textit{significant figures})$$

Note that exact numbers like 10mm in a centimeter have an infinite number of significant figures. We do not write the significant figures down and these numbers are not used in determining the number of significant figures to be used in your answer.

Significant Figures in Addition and Subtraction

To determine the number of significant figures in an addition or subtraction problem it is neccessary to look at the number with the *least* number of significant figures to the right of the decimal place. The number of decimal places will determine the number of significant figures to be used in the answer. For example, 29.4165 has 4 significant digits to the right of the decimal place and 234.65 has only 2. Therefore your answer will be rounded off to the 2nd digit after the decimal place.

$$29.4165+234.65=264.07$$

Uncertainty of Measurements

How precise the final answer will be is determined by the least precise measuring instrument used. For example if an object is measured with a set of calipers with a resolution to 0.001 inches the measurement would be recorded to the thousandth of an inch, 2.200 inches. 2.2 inches show it was measured with a device that only measures to 0.1 inches. 2.200 shows it was measured with a device that measures to 0.001 inches. Even though these measurements are the same the uncertainty of measurement is not.

Significant Figures in Measuring

A simple density calculation is used to illustrate the use of significant figures. *A*n object weighs 16.5 grams and has a volume of 9.3 milliliters. The formula for calculation is $density=\frac{mass}{Volume},(\rho=\frac{m}{V})$. By inserting the appropriate values we'll illustrate how this works.

$$density=\frac{16.5\text{g}}{9.3\text{mL}}=1.7\text{g}/mL\,(2\ significant\ \ figures)$$

In this example, 16.5g has 3 significant figures and 9.3mL has 2 significant figures. Using the rules of multiplying and dividing with significant figures, the answer would have the same number of significant figures as the *least* number in the calculation. Therefore, the answer should be reported using 2 significant figures, 1.7g/mL.

Let's try another density calculation. We have a rectangular rubber block and want to calculate the density to determine from what type of elastomer it is made. To calculate the volume of a rectangular rubber block I need to use the formula Length X Width X Height (LxWxH). I will substitute this for volume in the density formula.

$$density=\frac{mass}{(Length\,x\,Width\,x\,Height)}$$

A scale will be used to measure the mass and calipers to measure the dimensions of the block and calculate the volume. The scale has a resolution of 0.001 grams and the calipers have a resolution to 0.01mm. The results of the measurements are as follows:

Instrument	Measured Value
Scale with 0.001g Resolution	372.561g
Digital Calipers with 0.01mm Resolution	Length = 10.00mm Width = 10.00mm Height = 30.00mm

Note that the length, width and height values are recorded to 2 decimal places and the mass to 3 decimal places as these are the resolutions of instruments used and therefore want to show the uncertainty of measurements in the final answer. For example, if the measurement was recorded as 10mm, which has 2 significant figures, as opposed to 10.00mm, which has 4 significant figures, the measurement would not be expressing the proper uncertainty of measurement.

We want our density value in grams per cubic centimeter because the known density's of rubber is specified in grams per cubic centimeter not grams per cubic millimeter, it is therefore necessary to convert the length, width and height measurements from millimeters to centimeters. 10.00mm divided by 10 mm/cm = 1.000cm. Because 10mm/cm is an exact number with infinite number of significant figures, it's not used in determining the number of significant figures in the answer. Therefore, the number of significant figures is determined using measurements, which have 4 significant figures, so the final values can only have 4 significant figures, 1.000cm, 1.000cm and 3.000cm.

Let's plug in the values into the density formula and calculate the results using significant figures. Using the measured values in cm, the density is calculated:

$$density=\frac{3.697\text{g}}{(1.000\text{cm}\ X\ 1.000\text{cm}\ X\ 3.000\text{cm})}$$

$$density=\frac{3.697\text{g}}{(3.000\text{cm}^3)}$$

$$density=1.232\text{g}/cm^3\,(4\ significant\ \ figures)$$

The answer has 4 significant figures because the least number of significant figures in any number used is 4. Note that the numbers in the formula have the unit of measure which are also calculated out in the final answer. For example, in 1.000cm X 1.000cm X 3.000cm, the 3 "cm" are expressed as "cm^3". In the final value we have "grams" divided by "cm^3" expressed as "g/cm^3".

It is good practice to include significant figures in your calculations to show the uncertainty of your measurements so other know the precision at which you were able to measure. It is also good practice to always include the unit of measure in your values. These will be calculated out in your formula and used to determine the proper unit of measure in your final answer.